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Noncommutative hyper-Kähler structure for K3 surfaces

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Abstract

We apply the method of algebraic deformation to N-tuple of algebraic K3 surfaces. When $N = 3$, we show that the deformed triplet of algebraic K3 surfaces exhibits a deformed hyper-Kähler structure. The deformation moduli space of this family of noncommutative K3 surfaces turns out to be of dimension 57, which is three times that of complex deformations of algebraic K3 surfaces.

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1. Introduction

Noncommutative geometry [1] is now an integral part of string/M-theory [2]. Since the work of Connes *et al* [3] connecting the noncommutative torus [4, 5] and the T-duality in the M-theory context, various properties of noncommutative space itself such as noncommutative tori and their varieties have been a subject of intensive study [2, 6–8]. However, more interesting and complicated structures such as noncommutative orbifolds and noncommutative Calabi–Yau (CY) manifolds have been studied far less [9–14]. Also, not much is known about noncommutative spaces with complex structures. Only recently, have noncommutative tori with complex structures been studied [15–17].

In investigating the properties of noncommutative space with complex structure, the algebraic geometry approach seems to be a good fit. In [18], Berenstein *et al* initiated an algebraic geometry approach to noncommutative moduli space. Then applying this technique, Berenstein and Leigh [9] studied noncommutative CY threefolds; a toroidal orbifold $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ and an orbifold of the quintic in $\mathbb{CP}^4$, each with discrete torsion [19–23]. In their first example, they deformed the covering space in such a way that the centre of the deformed algebra corresponded to the commutative classical space, a CY threefold. In that process, the complex structure of the centre was also deformed as a consequence of the covering space deformation, and some part of the moduli space of complex deformations was indeed recovered. They could also explain the fractionation of branes at singularities from a noncommutative geometric viewpoint in the presence of discrete torsion. There, in order to be compatible with $\mathbb{Z}_2$ discrete torsion, the three holomorphic coordinates y_i which are the defining variables of the three elliptic curves of T^6, started to anticommute with each other.

In the commutative K3 case, the moduli space for the K3 space itself is known already (see for instance [24]), and even the moduli space for the bundles on K3 surfaces has been studied [25]. In [10], algebraic deformation of K3 surfaces has been studied in the case of the orbifold $T^4/\mathbb{Z}_2$. There, the work was carried out by considering deformation of the invariants of the K3 itself, unlike the deformation of the variables of the covering space as in [9].

In [26], this method was applied for the algebraic K3 case. Classically, the complete family of complex deformations of K3 surfaces is of dimension 20 inside which that of the algebraic K3 surfaces is of dimension 19 [24]. In [26], a 19-dimensional family of the noncommutative deformations of the general algebraic K3 surfaces was considered. The construction was similar to the Connes–Lott 'two-point space' construction of the standard model [27]. It was done by deforming a pair of algebraic K3 surfaces and was called 'two-point deformation'. It was further generalized to the N-point case by considering the deformation of N-tuple of algebraic K3 surfaces. In the N-point deformation, the dimension of deformation moduli turned out to be $19N(N-1)/2$ [26].

In this paper, we examine the N-point deformation method in the $N = 3$ case. Considering a 57-dimensional family of the noncommutative K3 surfaces, we show that the $N = 3$ case corresponds to a noncommutative deformation of the hyper-Kähler structure of K3 surfaces.

In section 2, we explain the method of N-point deformation for the algebraic K3 surfaces in detail for $N = 2$. In section 3, we show that for $N = 3$ this family of deformed noncommutative K3 surfaces exhibits deformed hyper-Kähler structures. In section 4, we give another interpretation in terms of Clifford algebras. In section 5, we conclude with a discussion.

2. Two-point deformation

In this section, we explain the method of N-point deformation [26] of algebraic K3 surfaces, specifically for the $N = 2$ case, by considering the 'two-point space' version of noncommutative deformation for a pair of algebraic K3 surfaces.

The N-point method was carried out by a direct extension of the algebraic deformation done for the $T^4/\mathbb{Z}_2$ case [10]. General algebraic K3 surfaces are given by the following form and with a point added at infinity:

$$y^2 = f(x_1, x_2). \tag{1}$$

Here f is a function with total degree 6 in x_1, x_2.

Now, we compare this with the Kummer surface, the orbifold of $T^4/\mathbb{Z}_2$ case [10]. There T^4 was considered as the product of two elliptic curves, each given in the Weierstrass form

$$y_i^2 = x_i(x_i - 1)(x_i - a_i) \tag{2}$$

with a point added at infinity for $i = 1, 2$. By the following change of variables, the point at infinity is brought to a finite point:

$$y_i \longrightarrow y_i' = \frac{y_i}{x_i^2} \qquad x_i \longrightarrow x_i' = \frac{1}{x_i}. \tag{3}$$

For algebraic K3 surfaces, we first consider a function with total degree 6 in complex variables u, v, w, for instance

$$F(u, v, w) = u^2v^3w + u^4v^2.$$

In a patch where the point at infinity of w can be brought to a finite point, dividing both sides by w^6 the above expression can be rewritten as

$$f(x_1, x_2) = x_1^2 x_2^3 + x_1^4 x_2^2$$

where $x_1 = \frac{u}{w}, x_2 = \frac{v}{w}$. The corresponding algebraic K3 surface is given by

$$y^2 = f(x_1, x_2) = x_1^2 x_2^3 + x_1^4 x_2^2. \tag{4}$$

Similarly, in a patch where the point at infinity of u can be brought to a finite point, we can re-express it as

$$y'^2 = f'(x'_1, x'_2) = x_1'^3 x'_2 + x_1'^2 \tag{5}$$

where $x'_1 = \frac{v}{u} = \frac{x_2}{x_1}, x'_2 = \frac{w}{u} = \frac{1}{x_1}$. Thus, in the case of the general algebraic K3, a point at infinity in one patch can be brought to a finite point in another patch by the following change of variables

$$y \longrightarrow y' = \frac{y}{x_1^3} \tag{6}$$

$$x_1 \longrightarrow x'_1 = \frac{x_2}{x_1} \qquad x_2 \longrightarrow x'_2 = \frac{1}{x_1}. \tag{7}$$

We now consider a noncommutative deformation of algebraic K3 surfaces. Following the same reasoning as in [10], we consider two commuting complex variables x_1, x_2 and two noncommuting variables t_1, t_2 such that

$$t_1^2 = h_1(x_1, x_2) \qquad t_2^2 = h_2(x_1, x_2) \tag{8}$$

where h_1, h_2 are commuting functions of total degree 6 in x_1, x_2. To be consistent with the condition that t_1^2, t_2^2 belong to the centre, one can allow the following deformation for t_1, t_2:

$$t_1 t_2 + t_2 t_1 = P(x_1, x_2). \tag{9}$$

Here the right-hand side should be a polynomial and free of poles in each patch. Thus, under the change of variables (7)

$$x_1 \longrightarrow x'_1 = \frac{x_2}{x_1} \qquad x_2 \longrightarrow x'_2 = \frac{1}{x_1}$$

t should be changed into

$$t_i \longrightarrow t'_i = \frac{t_i}{x_1^3} \qquad \text{for} \quad i = 1, 2. \tag{10}$$

This is due to the fact that t transform just like y in (6). Therefore, P transforms as

$$P(x_1, x_2) \longrightarrow x_1^6 P'\left(\frac{x_2}{x_1}, \frac{1}{x_1}\right). \tag{11}$$

This implies that P' should be of total degree 6 in x'_1, x'_2, at most. Interchanging the role of P and P' one can see that P should also be of total degree 6 in x_1, x_2.

The above structure was understood as follows. If condition (9) is not imposed, then there exist two independent commutative K3 surfaces. Once condition (9) is imposed, these two commutative K3 surfaces become a combined surface in which the two K3 surfaces intertwine with each other everywhere on their surfaces and become fuzzy. This seems to be similar to the two-point space version of the Connes–Lott model [27]. In the Connes–Lott model, every point of the space becomes fuzzy due to the one-to-two correspondence at each point in the space, where the two corresponding points at each classical location are fixed. On the other hand, the present case is similar to the relation between position x and momentum p in quantum mechanics at every point in the space. However, since the two copies of the classical space are combined to become a noncommutative space just like the Connes–Lott model, this construction was also called two-point deformation though its nature is a little different from that of Connes–Lott.

To count the dimension of the deformation moduli, one simply needs to count the dimension of the polynomials of degree 6 in three variables from (11) up to constant modulo projective linear transformations of three variables. Namely, $28 - 1 - 8 = 19$, where 28 is the dimension of polynomials of degree 6 in three variables and 1 and 8 correspond to a constant and $PGL(3, \mathbb{C})$, respectively.

3. Deformed hyper-Kähler structure

In this section, we consider the N-point deformation for $N = 3$. Following the method of two-point deformation in the previous section, we consider commuting variables x_1, x_2 and three noncommuting variables t_1, t_2, t_3. Here, each t_i^2 should belong to the centre and be a function of total degree 6 in x_1, x_2, such that

$$t_1^2 = h_1(x_1, x_2) \qquad t_2^2 = h_2(x_1, x_2) \qquad t_3^2 = h_3(x_1, x_2) \tag{12}$$

where h_1, h_2, h_3 are commuting functions of total degree 6 in x_1, x_2. To be consistent with the condition that t_i^2 belong to the centre, we can allow the following deformation for t_i:

$$t_i t_j + t_j t_i = P_{ij}(x_1, x_2) \qquad i, j = 1, 2, 3 \quad (i \neq j). \tag{13}$$

Here P_{ij} should be polynomials and free of poles in each patch. Thus, when we change from one patch to another, for instance under the change of variables (7) in the previous section,

$$x_1 \longrightarrow x_1' = \frac{x_2}{x_1} \qquad x_2 \longrightarrow x_2' = \frac{1}{x_1}$$

t_i should be changed into

$$t_i \longrightarrow t_i' = \frac{t_i}{x_1^3} \qquad \text{for} \quad i = 1, 2, 3. \tag{14}$$

This is due to the fact that t_i transform just like y in (6) in the previous section under the above change of patches. Therefore, under the above change of variables P_{ij} transform as

$$P_{ij}(x_1, x_2) \longrightarrow x_1^6 P_{ij}'\left(\frac{x_2}{x_1}, \frac{1}{x_1}\right). \tag{15}$$

By the same reasoning as in the two-point deformation case, one can see that each P_{ij} is of total degree 6 in x_1, x_2, at most. It is not difficult to show that one can also get the same conclusion for different changes of patches. Here, conditions (18) for t_i^2 represent the different complex deformations of K3 surfaces, and its moduli space is of complex dimension 57. Condition (19) provides a characteristic of noncommutativity for otherwise three separate commutative (algebraic) K3 surfaces given by (18). Since each P_{ij} is a polynomial of total degree 6 in x_1, x_2, condition (19) makes the moduli space of the above noncommutatively deformed K3 surfaces be of complex dimension 57, the same as the moduli dimension of complex deformations that we explained above and this is exactly three times that of the commutative algebraic K3 surfaces. This is different from the commutative hyper-Kähler K3 case, in which the moduli space is of real dimension 58 as we will discuss below. Then, what is the relationship between our newly constructed noncommutative K3 surface and the hyper-Kähler structure of commutative K3 surfaces?

Before we address this question, we first review the property of the moduli space $\mathcal{M}$ of Ricci-flat metrics on a K3 surface S. If a given metric g satisfies $g(Jv, Jw) = g(v, w)$ for any tangent vector v, w, then we say that the metric g is compatible with the complex structure J. If the two-form $\Omega(\cdot, \cdot) = g(J\cdot, \cdot)$ is closed, then it is called a Kähler metric and Ω is called a Kähler form. Any given Ricci-flat metric g induces a Hodge $*$ operator on $H^2(S, \mathbb{R}) \cong \mathbb{R}^{3,19}$ by which $H^2(S, \mathbb{R})$ can be decomposed as a direct sum of two eigenspaces, self-dual part (eigenvalue 1) of dimension 3 and anti-self-dual part (eigenvalue -1) of dimension 19. The self-dual part is positive definite with the integration on S after wedge product, so that the moduli space of Ricci-flat metrics is locally isomorphic to $(O(3, 19)/O(3) \times O(19)) \times \mathbb{R}_+$. This is because $H^2(S, \mathbb{R})$ has the intersection form (3,19) and the parameter of the scaling of the metric is $\mathbb{R}_+$. So the real dimension of $\mathcal{M}$ is $3 \times 19 + 1 = 58$.

We can also understand this in a different setting. Let $\mathcal{N} = \{(J, \Omega) \mid \Omega$ is a Kähler form in the K3 surface with the complex structure $J\}$. Then the real dimension of $\mathcal{N}$ is equal to the real dimension of the moduli space of complex structures plus the real dimension of Kähler forms, which is $40 + 20 = 60$. We can define a map Φ from $\mathcal{N}$ to $\mathcal{M}$ as follows:

$$\Phi((J, \Omega)) = g \qquad \text{such that} \quad g(\cdot, \cdot) = \Omega(\cdot, J\cdot).$$

Then it is onto but not one-to-one. The inverse image of g by Φ is $\mathbb{P}^1$. So,

$$\dim_{\mathbb{R}} \mathcal{M} = \dim_{\mathbb{R}} \mathcal{N} - 2 = 60 - 2 = 58.$$

Now, we define the hyper-Kähler structure on S. In the first setting, for the given Ricci-flat metric g, the self-dual part Λ^+ is a three-dimensional real vector space consisting of vectors whose self-intersection is positive. With any compatible complex structure J to g, we associate Ω which is a vector in Λ^+ and is a (1, 1) form. Then real (2, 0) and (0, 2) forms in Λ^+ are exactly orthogonal to Ω. Different compatible structures J to g correspond to different unit vectors in Λ^+, and they form S^2 isomorphic to $\mathbb{P}^1$, inverse of $\Phi^{-1}(g)$. Here we choose three orthogonal unit vectors $\Omega_1, \Omega_2, \Omega_3$ in Λ^+ such that the corresponding complex structures J_1, J_2, J_3 satisfy the relation $J_i J_j = \epsilon_{ijk} J_k$ for $i, j, k = 1, 2, 3$. This is called a hyper-Kähler structure on S.

Now we return to our question of the connection between our $N = 3$ construction and the hyper-Kähler structure of K3 surfaces. When P_{ij} all vanish in (19), the t_i $(i = 1, 2, 3)$ in (19) satisfy the same relation as the complex structures J_i $(i = 1, 2, 3)$ in the case of the commutative hyper-Kähler K3 surfaces, and t_i actually correspond to the complex structures of the commutative K3 surfaces. Here, if we consider just one of the t_i and disregard the other two t_i for a moment, then the t_i represents a family of commutative algebraic K3 surfaces whose moduli dimension is of complex dimension 19. On the other hand, when all three t_i are present but all the P_{ij} vanish in (19), then the t_i are not independent of each other like the J_i of the commutative hyper-Kähler K3 case. However, when all P_{ij} do not vanish and are independent of each other, t_i become all independent and the moduli dimension becomes three times larger than that of each piece represented by one of the t_i. Thus, the space becomes noncommutative under condition (19) provided that all P_{ij} do not vanish and are independent of each other. Therefore, we can regard our new noncommutative K3 surfaces as deformed hyper-Kähler K3 surfaces, since t_i satisfying (18) and (19) with vanishing P_{ij} do admit the hyper-Kähler structure [24, 28].

This we can see by redefining t_j $(j = 1, 2, 3)$ as $t_j = \mathrm{i}\sqrt{h_j(x_1, x_2)}\hat{t}_j$ for $j = 1, 2, 3$. Then, in terms of $\hat{t}_j$, (18) and (19) become

$$\hat{t}_1^2 = -1 \qquad \hat{t}_2^2 = -1 \qquad \hat{t}_3^2 = -1 \tag{16}$$

and

$$\hat{t}_i\hat{t}_j + \hat{t}_j\hat{t}_i = -P_{ij}(x_1, x_2)/\sqrt{h_i h_j} \qquad i, j = 1, 2, 3 \quad (i \neq j). \tag{17}$$

Recall that the quaternion structure of the hyper-Kähler structure of K3 surfaces can be expressed as

$$J_i^2 = -1 \qquad J_i J_j = \epsilon_{ijk} J_k \qquad i, j = 1, 2, 3$$

and thus

$$J_i J_j + J_j J_i = 0.$$

Comparing with this we see that newly defined $\{\hat{t}_j\}$ exhibit a deformed hyper-Kähler structure for K3 surfaces.

The above construction of noncommutative hyper-Kähler K3 surface is a little different from that defined in [28, 29] whose T_i ($i = 1, 2, 3$) operators perform a similar role to our t_i. In [28, 29], the commutation relation among T_i was not deformed, it remains the same as that of J_i in the commutative hyper-Kähler K3 case. However, in their definition there exist extra anticommuting operators which provide holomorphic structures, and we wonder whether these additional anticommuting operators could make the two constructions equivalent.

4. K3 surface embedded in a Clifford variety

We now give another interpretation of these phenomena in the context of Clifford algebras. The equations for t_i are

$$t_1^2 = h_1(x_1, x_2) \qquad t_2^2 = h_2(x_1, x_2) \qquad t_3^2 = h_3(x_1, x_2) \tag{18}$$

$$t_i t_j + t_j t_i = P_{ij}(x_1, x_2) \qquad i, j = 1, 2, 3 \quad (i \neq j). \tag{19}$$

Here we can understand t_i as the homogeneous variables of tangent space of P^2 and those h_i and P_{ij} make a metric on P^2. Locally, these are just Clifford algebras as shown in the previous section. We obtain a variety defined with those h_i and P_{ij} in a weighted projective variety $P^2(1, 1, 1) \times P^2(3, 3, 3)$. The first $P^2(1, 1, 1)$ represents the $P^2(u, v, w)$, which is the base space. (We used x_1, x_2 in a local chart.) The next $P^2(3, 3, 3)$ represents the $P^2(t_1, t_2, t_3)$ of the tangent space of P^2. So, the above equations define a subvariety in the weighted projected variety, which might be called a Clifford variety of P^2. It is interesting to see that three K3 surfaces define a K3 surface in a Clifford variety.

5. Discussion

We deformed the algebraic and hyper-Kähler K3 surfaces in both noncommutative and complex directions. In the deformation of hyper-Kähler structure, we introduced three noncommuting variables which correspond to three copies of commutative K3 surfaces and at the same time represent three different complex structures of K3 surfaces. Before deformation, we make these three variables have the same relation as the three complex structures J_i ($i = 1, 2, 3$) of hyper-Kähler K3 in which J_i possess the quaternionic structure and anticommute with each other. Here, one may wonder whether the deformation condition (19) could also be satisfied with commuting variables when the polynomials P_{ij} do not vanish. That is possible, but the consequences are totally different depending on whether these variables are commuting or noncommuting ones. When they are commuting variables, P_{ij} in (19) are not independent and they can all be expressed in terms of h_i ($i = 1, 2, 3$) functions in (18). Thus, there are only complex deformations and no noncommutative deformations. On the other hand, when these variables are noncommuting ones and P_{ij} are nonvanishing, then P_{ij} in (19) are all independent of h_i functions. Hence, we have both complex deformations from h_i and noncommutative deformations from P_{ij}. And since our construction is a deformation from the commutative hyper-Kähler structure of K3 surfaces in the noncommutative direction, we end up with a noncommutative hyper-Kähler structure for K3 surfaces.

About the moduli dimension of our noncommutative hyper-Kähler structures for K3, we still do not have a clear understanding of how ours is related to the commutative one. In the commutative case, it has real moduli dimension 58 as we explained before. On the other hand, our noncommutative hyper-Kähler structure has complex moduli dimension 57. Apparently, ours is exactly twice that of the commutative one, once we disregard the parameter of overall scaling in the commutative case. Thus, if we complexify the metric moduli, then it seems that we can fill the gap. In the construction of hyper-Kähler K3 in [28, 29], there exist a set of anticommuting operators providing the complexification. However, we do not have the corresponding variables in our construction as we mentioned briefly at the end of the last section. Since the commutation relation of the operators representing the complex structures in those works is not deformed, unlike in our construction, there is some possibility that our noncommuting variables may also possess the property of these anticommuting operators in [28, 29]. We will leave this investigation for our future work.

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